Drawing de Bruijn Graphs

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The simplest kind of de Bruijn graph, in which the number of nodes is a power of two, has, at each node, two edges directed out and two edges directed in.

When the number of nodes is 2^n, we can label them with the integers from 0 to $2^n - 1$ and notice that they correspond to all the numbers expressible with n bits (n binary digits). Out from the node labeled x the two directed edges go to the nodes labeled $2x$ and $2x + 1$. If $2x$ or $2x + 1$ is bigger than $2^n - 1$, just subtract 2^n from it to bring the number back into the range from 0 to $2^n - 1$.

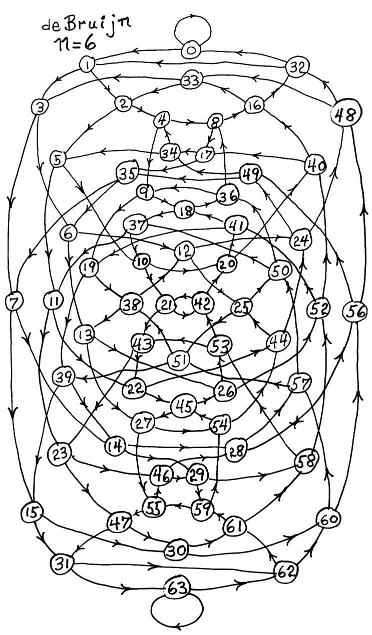

Figure 1. $n = 6$.

There is reason to think that the picture for $n = 6$ may fill a gap in the existing literature, because Hal Fredricksen told me he had not seen it in any book. Pictures up to $n = 5$ can be found in the classic "Shift Register Sequences" by Solomon W. Golomb, available from Aegean Park Press.

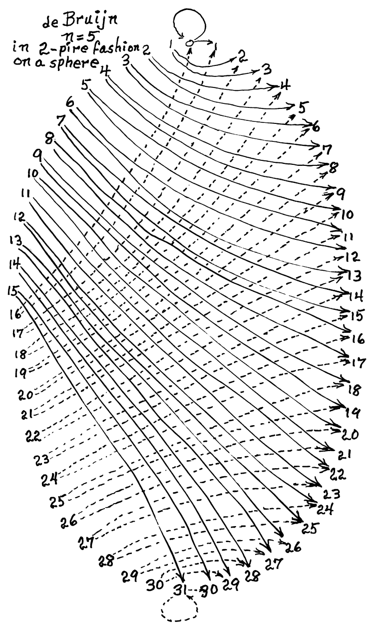

Figure 2. $n = 5$.

Bigger de Bruijn graphs get really hairy. The picture for $n = 5$ suggests 2-pire fashion[1] for drawing the de Bruijn graph on the sphere without any lines crossing each other. The scheme would be to put 0 on the North pole, $2^n - 1$ on the South pole, the numbers from 1 to $2^n - 2$ in order going south down the Greenwich Meridian, and down the international date line. Edges out from 0 to $2^{n-1} - 1$ on the meridian go east to the dateline, while edges out from 2^{n-1} to $2^n - 1$ go west.

[1]In 2-pire fashion each node of the graph is allowed to appear in two places in the picture. The earliest reference I know of to the *m*-pire (empire) problem is P. J. Heawood's 1891 paper in the *Quarterly Journal of Mathematics*.

See *Scientific American,* February 1980, Vol. 242, No. 2, "Mathematical Games," by Martin Gardner, pp. 14–22.