Beautiful But Wrong: The Floating Hourglass Puzzle

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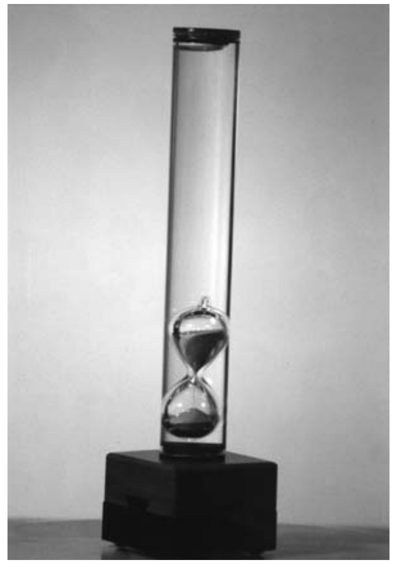
The Beginning

One of the problems in Martin Gardner's August 1966 "Mathematical Games" column was The Floating Hourglass.

> An unusual toy is on sale at a Paris shop: a glass cylinder, filled with water, and at the top an hourglass floats. If the cylinder is inverted a curious thing happens; the hourglass remains at the bottom of the cylinder until a certain quantity of sand has flowed into its lower compartment. Then it rises slowly to the top. It seems impossible that a transfer of sand from top to bottom of the hourglass would have any effect on its overall buoyancy. Can you guess the simple *modus operandi*?

I gave the problem some thought but couldn't come up with any good theory. I assumed it had to do with some law that I had forgotten since high school. The next month, when the answer came, I was delighted. It was so simple, so absurdly obvious, that I not only *could* have thought of it myself, I *should* have. The effect was like seeing a good magic trick or hearing a good joke.

I read Martin Gardner's columns religiously, and corresponded with him occasionally from 1963 on, as a college student, as a graduate student, and then as an editor of *Psychology Today*. In 1978, in the months before a new science magazine, Omni, was to be launched, I had the pleasure of finally meeting the Master Explainer. I was going to write a column on "Games" so I made a pilgrimage to Hastings-on-Hudson to visit the Master. There on a shelf was the infamous Floating Hourglass itself. I could finally try out the curious toy I had read about so many years before. I turned it over and the hourglass stayed at the bottom, just like Martin said it would.

At the 1991 Puzzle Collectors Party in Los Angeles, I saw my second Floating Hourglass and knew I could finally write about it, since I only published puzzles in *Omni* that I knew my readers could find. Tim Rowett had brought one from England, made by Ray Bathke of London. I immediately ordered some. My September 1992 column introduced the Floating Hourglass 25 years after I first heard about it. I asked my clever readers to submit theories to explain it. The results appeared in the January 1993 issue.

When Martin allowed me to look through his files, I found a thick folder on the Hourglass, a treasure trove of letters and drawings. For years I have itched to tell this story, but no magazine article could possibly contain it. This book finally gives me the chance to tell the history of the Floating Hourglass Puzzle.

The First Theories

Piet Hein, the Danish sculptor/inventor (the Soma Cube, the Super Egg) and poet/artist (Grooks), had visited Martin in early 1966. Hein told him about a toy he had seen in the Paris airport. He didn't bring one back, but his description was clear enough; Gardner knew how it must work and wrote about it in his August and September columns without even having seen one. Martin's theory relied on friction between the glass and the cylinder, but Piet wasn't convinced. He thought the inside of the cylinder he had seen was too smooth to offer much resistance. He felt there must be something more, something to do with the falling sand.

Hein believed the impact of the sand grains hitting the bottom of the glass exerts a downward force, an 'effective change in mass': "The hourglass is heavier while the sand is falling," Hein wrote Gardner on September 16, 1966. "Imagine if the hourglass were opaque and you didn't know it were an hourglass at all. There it stands at the bottom and changes its weight!...What keeps the hourglass down is the falling of the sand, not the amount of sand that has fallen or is left. The hourglass rises not because there is little sand left in the top chamber, but because the rate of falling sand has decreased. This seems to solve the whole problem."

Note that he wrote all this after the September *Scientific American* was out. He knew of Gardner's "answer," but he also knew that Martin had never examined an actual hourglass sample. For him this meant that the final proof was not yet in. Until you could see and touch one, break it open or learn how it is made, all theories were valid contenders. In the absence of knowing the truth, the best criterion for a theory is its beauty. And Martin had already admitted that Piet's impact theory was beautiful.

Hein was obsessed with hourglasses. He drew a cartoon of himself on an elevator with Einstein, pondering an oversized hourglass. He designed an hourglass-powered perpetual motion machine and created a fantasy ocean full of bobbing hourglasses. Since the glasses change their weight whenever the sand falls inside, Hein issued a mock warning: When mailing hourglasses, don't weigh the package while they're running, or you'll have to pay a higher postage.

A Painful Paradigm Shift

Just a couple of days after writing the letter and drawing the cartoon, Hein had an agonizing experience in Milan, Italy. In a shop there he saw a double-glass: two cylinders side by side, a floating hourglass in one and a sunken hourglass in the other. When turned over, the glasses stay in place at first and then one rises while the other sinks. He knew immediately that his impact theory was doomed. A "sunken" hourglass that stays in place at the top of a tube can't be explained by sand grains falling in the opposite direction.

Hein tried the double-glass in the shop a few times, just enough to see that it worked. "That was all I wanted to know," he wrote "and all that was needed to make my intellectual headache come back much worse than the first time.” Hein was forced to make a paradigm shift, and he found it painful. He knew that the sinking glass directly refuted his theory, but he didn’t buy one to take home. “This is not a question of fumbling one’s way to a solution, but of thinking,” he explained. “I couldn’t think of anything else but the principle. It really hurt.”

He couldn’t bear to abandon his pet idea *completely*. He rationalized in jest: “My theory works, with the exception that the sand was running downward in both hourglasses, I must admit, but it would be easier to explain the symmetry of the phenomenon if it were falling upward in one of them!”

Hein acknowledged Gardner’s appreciation of beauty in a theory. “I am glad you think my explanation is beautiful,” he wrote. “So do I, but let us be honest and not rate it any lower just because it is false. I admit, being false makes it less right. But it does not make it any less beautiful.”

When Piet Hein left Milan on the afternoon of September 21, he was somber, “somewhat worried and depressed on behalf of my beautiful theory.” Then, as the plane soared over Mont Blanc, Hein had a sudden crystallizing insight - “At an altitude of 8500 meters, I found myself in possession of the solution.” He later wrote about the “Aha” experience. “When the Alps in the lite of the (superelliptic) setting sun and the growing, exactly half moon (D for dynamics) were left behind us and all was dark, there was a lull in my mind, a *tabula rasa.* And on that tablet lay, like a small fish on the center of a huge platter, the solution of the hourglass mystery. My dear Watson, it’s very simple.”

The Solution Is in the Solution

The key to the puzzle isn’t *in* the glass but *around* it, Hein realized, not in the falling of the sand but in the flowing of the liquid. There must be two liquids of different specific weights that don’t mix completely, and are indistinguishable in color and transparency.

When the cylinder is inverted, the heavier liquid is on top pushing down. Only when enough of it has seeped down does the glass begin to rise. The falling sand is just for misdirection, but what a clever ruse it is. The hourglass puzzle is caused by an hourglass effect, but the relevant displacement is in the liquid, not the sand. Hein was awed by the brilliance of it all, “I should like to meet the person who invented this effect and designed it so as to hide the solution so elegantly for us,” he wrote on September 22.

Gardner’s reply on October 5th was a gentle letdown: “You said you would like to meet the clever fellow who thought of the ‘two-liquid’ principle.

Well, all you have to do is shake your own hand. You are the inventor. The principle is simply too clever to be true."

The Hourglass Letters

While Hein was having his epiphany in Europe, Gardner was getting letters in response to the September issue. One man wrote that he owned one of the Paris cylinders, but reported that his glass sometimes floated and sometimes sank, perhaps depending on the temperature.

On September 6th, Albert Altman wrote from the U.S. Naval Ordnance Laboratory at Silver Spring, Maryland:

> Another solution to the hourglass science teaser is that the momentum carried by the falling sand causes the hourglass plus sand to weigh more than its static weight by an amount $\mu\sqrt{2gh}$, where μ is the rate of the flow of the sand, g the acceleration of gravity, and h the height through which the sand has fallen. The height decreases due to the buildup of sand on the bottom of the hourglass and at a critical value the net force on the hourglass acts upward and rises.

Gardner was beginning to wonder if his friction explanation told the whole story. Could temperature and sand impact also be factors? Would he have to print a correction? He replied to Altman on September 12th:

> I am embarrassed to admit that your explanation may be right. I have not yet seen the toy; having relied (unfortunately) on an account given to me by a friend who examined the toy in Paris, but did not bring one back with him. It is possible that one version of the toy works on the principle you mention, and the other on the principle I suggested, or perhaps still another one. In short, at this point I am hopelessly confused.

Hopelessly confused? Martin Gardner?! That is something that surely doesn't happen very often, and it didn't last long. A letter dated September 29 came from Walter P. Reid, also from the U.S. Naval Ordnance Lab. "I am writing to put your mind at ease (on the impact theory posed by Altman), and to suggest that you not publish a correction. I am sure that your explanation was correct." Reid went on to show mathematically how the impact of sand hitting the glass' bottom is exactly balanced by the loss of the sand's mass while it is in free fall. Reid later adapted his letter for publication and his short article "Weight of an Hourglass" appeared in *American Journal of Physics,* 35(4), April, 1967. This remains, to my knowledge, the only scientific writing on the subject. Gardner cites it in the hourglass puzzle reprinted in *Mathematical Circus.*

Hein and the Horse's Mouth

All was settled for a few months, but then in the spring of 1967 the glass rose again. Hein wrote that he went back to the shop in Paris where he first saw the glass, and tracked down the maker. He turned out to be a Czechoslovakian glassblower named Willy Dietermann, and he confirmed Hein's two-liquid theory. When Hein asked about the liquid, Mr. Dietermann explained it was 90 to 95% water and 5 to 10% a combination of alcohol and formic acid.

Gardner wrote to Dietermann but received no reply. He decided he had to write a correction after all. Set into type and slated to appear in a summer 1967 column was this recreation of the moment of truth: "When Hein stated his two-liquid theory, the inventor staggered as if struck. It was the first time his secret had been guessed: two liquids of different density, but which do not separate completely, so there is no visible division between the two. A slight difference in refractive power is concealed by the curvature of the glass cylinder. The glasses do tip to the side when inverted, but that is just a holding operation until the liquids have had time to change places. The main principle is the liquid hourglass effect, completely invisible, which eventually sends one hourglass up, the other down."

Proof Positive

By this time, Gardner had his own hourglass tube. He loaned it to C.L. "Red" Stong, the original "Amateur Scientist" columnist of *Scientific American.* In his lab, Stong shone polarized light through the cylinder and found that the refractive index of the liquid did not change from top to bottom. He concluded the liquid inside was homogenous. He also reported it had a freezing point of eight degrees below zero.

Meanwhile, Gardner was playing amateur scientist himself. He bought a cheap egg timer, freed the glass from the wooden frame, and found a transparent cylinder into which it would fit. "I filled the cylinder with water," he wrote to Hein, "then I wrapped copper wire around the middle of the hourglass. By snipping off the ends of the wire I was able to make its weight such that it slowly rises in the cylinder. It worked just like the big version."

Sanity was restored and the correction was never printed. We may never know what Hein got from the "horse's mouth." Perhaps Dieterman was leading him along, or perhaps there was an honest misunderstanding about the use of different liquids. There certainly was plenty of room for misinterpretations. The Dane and the Czech had to speak German because Dieterman knew no Danish or English, and his German was better than his French, and Hein's French was worse than his German.

Final Thoughts

What fascinated me about the hourglass puzzle was how it led a mind like Piet Hein's to come up with such brilliantly incorrect theories. They may have been wrong, but they were creative products of human thought, and deserved to be prized for that alone. Let others measure a refractive index or a freezing point, Hein wanted to think the problem through. He wanted to search for alternate, beautiful explanations. He wanted to expand his "perpendicular thinking."

I received over 1200 letters about the hourglass after publishing the puzzle in *Omni*. As I read them, sorting them into different piles, I found the largest single category was always the "correct" theory. This proportion stays at about 40% with each new batch of mail. The other 60% broke down into about 15 different theories.

About 40% of those readers with incorrect answers cited heat as a factor in the hourglass's behavior. Falling sand generates heat, they said. Some argued that this warms the surrounding liquid so the hourglass stays down until the liquid cools again; others, that the hourglass floates up with the pocket of warm liquid surrounding the glass's neck. But most in this category thought the heat warms the *air* in the glass, making it expand slightly and then rise.

More than 50 readers thought that the hourglass was flexible. Some reasoned that when the sand presses down from the top, the hourglass widens and wedges itself into the cylinder. Others decided that the hourglass is flexible only at the ends. "The top and bottom of the hourglass are so thin as to sag under the weight of the sand," wrote B. G. of Los Altos Hills, California. "When enough sand falls into the bottom chamber, it 'bubbles' the bottom end out, increasing the hourglass's volume," reasoned D. Q. of Richmond Hill, Ontario, Canada.

Many correspondents blamed the "impact" of falling sand for keeping the glass down. Some even used mathematical formulas to show how much force a sand grain exerted, first on the bottom of the glass and later on a mound of other sand grains. The theory may be correct, but the calculations have to consider the amount of time each grain of sand is falling and weightless; the two tiny opposing forces exactly cancel each other out. Movements within the system don't alter the weight of the system.

Another line of reasoning put the emphasis on the liquid: "The solution is in the solution!" wrote H. W. of Coweta, Oklahoma. If the liquid is naturally cooler and denser at the bottom, then the denser liquid is at the top when the tube is turned over. It eventually seeps down below the hourglass and buoys it up.

A surprising number thought it was all an illusion. "It just takes a long time for the hourglass to get started," perhaps because the liquid is very viscous, wrote one reader. "The hourglass, *as a system,* is rising from the moment the column is inverted," argued P. T. of Glendale, California. They concentrated on the air bubble that constantly rises, first in the hourglass and then in the tube.

Many believed the air at the *top* of the hourglass lifts it to the top of the tube. "When enough air reaches the top chamber and exerts its pressure there, the hourglass begins to rise," wrote T. H. of Chapel Hill, North Carolina. About 4% of those who wrote in thought that the shape of the hourglass affected its buoyancy. When the air is in the bottom half, the water below the glass can push up only on the circular end of the glass. When the air rises to the top half, water can push up all around the inverted cone, a greater surface area. "It's the same principle that causes a snow cone to pop out of its cup when you squeeze the bottom," explained D. A. N. of Tillamook, Oregon.

I promised copies of the book *Omni Games* to the five "most interesting" entries. Correct answers to this puzzle aren't very interesting because they're all virtually alike. Therefore, I awarded books for *incorrect* answers only:

(1) Pete Roche of Chicago foresaw this and sent both a correct theory and this incorrect alternate: "The hourglass has a small clasping mechanism at each end. The momentum of rising provides enough energy to engage the mechanism as it reaches the top of the cylinder, while the weight of the falling sand is required to release the mechanism."

(2) John J. Gagne of Eglin Air Force Base, Florida, supposed that sand blocked the hole, and air in the bottom of the hourglass became compressed until "a jet of air shoots into the top half of the hourglass, imparting just enough lift to overcome inertia and start the glass moving up."

(3) "The hourglass is composed of a flexible material such as Nalgene," wrote Timothy T. Dinger, Ph.D., and Daniel E. Edelstein,

Ph.D. The two share a prize for having the confidence to submit their incorrect theory on company stationery: IBM's Thomas J. Watson Research Center in Yorktown Heights, New York.

(4) Cliff Oberg of Clarkdale, Arizona, theorized that the tube's caps are hollow and that the fluid must flow from the tube through a hole into the cap below before the glass can rise.

(5) Finally, a theory about the density gradient of the liquid was signed "Bob Saville, Physics Teacher, Shoreham-Wading River High School, Shoreham, New York." He added, "P.S. If this is wrong, then my name is John Holzapfel and I teach chemistry." The fifth book, therefore, goes to Holzapfel.

The "correct" answer, as Martin Gardner wrote it in 1966:

> When the sand is at the top of the hourglass, a high center of gravity tips the hourglass to one side. The resulting friction against the side of the cylinder is sufficient to keep it at the bottom of the cylinder. After enough sand has flowed down to make the hourglass float upright, the loss of friction enables it to rise.