

# The Nine Color Puzzle

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The nine color puzzle consists of a tricube, with each cube a different color, and twelve different dicubes with each cube of a dicube a different color. Altogether there are three cubes each of nine different colors. The object of the puzzle is to assemble the pieces into a cube with all nine colors displayed on the six faces. A typical set is shown below in Figure 1, where each number represents a color. Typical solutions are shown at the end of this paper.

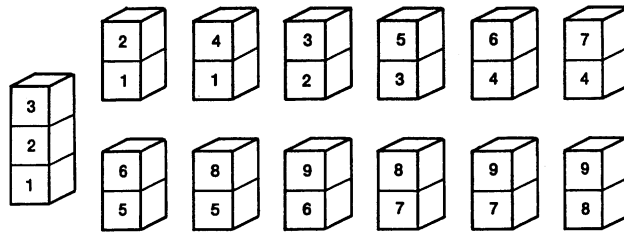


Figure 1.

According to Reference 1, the nine color puzzle was first introduced during 1973, in Canada, as “Kolor Kraze.” I learned of the puzzle in 1977 from Reference 2, and, using the name “Nonahuebes,” included it among the puzzles I produced in New Zealand under the aegis of Pentacube Puzzles, Ltd. I was intrigued by the puzzle’s characteristics but did not, until recently, analyze it thoroughly.

This puzzle is of particular interest because it is of simple construction, has an easily understood objective, has numerous variations and solutions, is a manipulative puzzle, and requires logical decision-making. Included in its analysis are the concepts of transformations, parity, backtracking, combinatorics, ordered pairs, sets, and isomorphisms. A complete analysis requires

the use of a computer, but after reading this article it should not be a difficult exercise for a moderately proficient programmer to verify the results obtained.

The first observation noted is that the cube can be assembled with the tricube along the edge or through the center of the cube, but not in the center of a face. A proof is given in Section 2.

The second observation is that for a cube to be assembled with the nine colors on each face, none of the nine planes may contain two cubes of the same color. A proof is given in Section 3. Thus, when two cubes of the same color are in position, the location of the third cube is determined. Since during the course of trying to solve this puzzle a conflict often occurs, this rule then tells the experimenter to backtrack.

The third observation, obtained after some experimentation, is that there are numerous solutions, some with the tricube on the edge and some with the tricube through the center of the cube. The question then arises: How many solutions are there?

Most intriguing about this puzzle, and the most difficult aspect to investigate was the fourth observation: the color combinations need not be as shown in Figure 1. The previous question now becomes more interesting. How many solutions are there for each color combination? And how many color combinations are there?

This paper addresses these last two questions. The individual dicubes can be colored thirty-six different ways. Twelve of these can be selected in  $1,251,677,700$  ( $36!/12!24!$ ) different ways of which, as determined in Section 4, only 133,105 of these combinations meet the three-cubes-of-each-color requirement. Since one person can assign the fourth and fifth colors to orange and red and another person to red and orange, it becomes apparent that most of the 133,105 combinations are isomorphisms. It becomes necessary, then, to separate these cases into disjoint sets of isomorphisms. Fortunately, as is shown in Section 5, only thirty dicubes are required, and only 10,691 combinations need to be sorted into disjoint sets.

The method of sorting the 10,691 combinations into 148 disjoint sets is described in Section 6, in which the number of isomorphisms using the set of thirty-six dicubes is also determined.

The final part of this analysis was to determine the number of puzzle solutions for each disjoint set of dicube combinations. This was achieved through the use of a computer program; the results are shown in Table 1.

Table 1.

| Isomorphisms |        | Number of Solutions |     |          |        |         |      |        |
|--------------|--------|---------------------|-----|----------|--------|---------|------|--------|
| No.          | Name   | dicubes             |     | straight |        | L shape |      |        |
|              |        | 36                  | 30  | Edge     | Center | Corner  | Edge | Center |
| 1            | .:Zlyz | 60                  | 60  | 0        | 0      | 0       | 0    | 0      |
| 2            | .:Zppp | 10                  | 10  | 0        | 0      | 0       | 0    | 0      |
| 3            | .COeyz | 360                 | 180 | 8        | 0      | 16      | 16   | 0      |
| 4            | .DOeyz | 720                 | 312 | 20       | 0      | 12      | 0    | 4      |
| 5            | .DZlyz | 1439                | 624 | 96       | 4      | 80      | 6    | 15     |
| 6            | .DZpez | 720                 | 312 | 198      | 12     | 83      | 10   | 16     |
| 7            | .DZppp | 360                 | 156 | 428      | 24     | 208     | 8    | 36     |
| 8            | /90eyz | 180                 | 60  | 16       | 0      | 38      | 32   | 0      |
| 9            | /:0eyz | 360                 | 120 | 80       | 4      | 24      | 4    | 4      |
| 10           | /:Zlyz | 720                 | 240 | 182      | 8      | 50      | 16   | 24     |
| 11           | /:Zpez | 360                 | 120 | 124      | 4      | 38      | 44   | 16     |
| 12           | /:Zppp | 180                 | 60  | 408      | 0      | 186     | 32   | 64     |
| 13           | /COeyz | 360                 | 72  | 0        | 0      | 6       | 4    | 0      |
| 14           | /DXeyz | 720                 | 132 | 18       | 0      | 4       | 2    | 0      |
| 15           | /DZlyz | 720                 | 132 | 72       | 0      | 32      | 12   | 0      |
| 16           | /DZpez | 1439                | 264 | 41       | 0      | 40      | 9    | 0      |
| 17           | /DZppp | 360                 | 66  | 80       | 0      | 117     | 28   | 0      |
| 18           | /MNeyz | 60                  | 8   | 0        | 0      | 0       | 0    | 0      |
| 19           | /MOeyz | 720                 | 96  | 8        | 0      | 34      | 8    | 16     |
| 20           | /MXeyz | 720                 | 84  | 32       | 0      | 30      | 12   | 12     |
| 21           | /MYeyz | 720                 | 84  | 26       | 0      | 18      | 6    | 4      |
| 22           | /MZlyz | 1439                | 168 | 62       | 2      | 70      | 21   | 23     |
| 23           | /MZpez | 1439                | 168 | 41       | 3      | 35      | 15   | 16     |
| 24           | /MZpez | 1439                | 168 | 51       | 4      | 57      | 17   | 20     |
| 25           | /MZppp | 720                 | 84  | 68       | 4      | 73      | 26   | 14     |
| 26           | /NXeyz | 720                 | 96  | 8        | 0      | 36      | 12   | 12     |
| 27           | /NYeyz | 1439                | 192 | 40       | 2      | 64      | 17   | 24     |
| 28           | /NZmyz | 720                 | 96  | 8        | 0      | 10      | 2    | 0      |
| 29           | /NZnpz | 1439                | 192 | 53       | 2      | 53      | 11   | 17     |
| 30           | /NZpez | 720                 | 96  | 36       | 4      | 48      | 24   | 32     |
| 31           | /NZpfp | 1439                | 192 | 10       | 0      | 49      | 34   | 23     |
| 32           | /Oblyz | 1439                | 168 | 25       | 1      | 81      | 24   | 16     |
| 33           | /Obmyz | 1439                | 168 | 98       | 5      | 65      | 18   | 19     |
| 34           | /Obofz | 720                 | 84  | 6        | 0      | 18      | 6    | 8      |
| 35           | /Obpez | 1439                | 168 | 124      | 3      | 53      | 8    | 17     |

| Isomorphisms |         | Number of Solutions |     |          |        |         |      |        |
|--------------|---------|---------------------|-----|----------|--------|---------|------|--------|
| No.          | Name    | dicubes             |     | straight |        | L shape |      |        |
|              |         | 36                  | 30  | Edge     | Center | Corner  | Edge | Center |
| 36           | /Obppp  | 720                 | 84  | 38       | 2      | 104     | 46   | 20     |
| 37           | /Oceyz  | 720                 | 84  | 18       | 0      | 46      | 6    | 8      |
| 38           | /Ocmyz  | 1439                | 168 | 34       | 3      | 27      | 12   | 2      |
| 39           | /Ocnpz  | 1439                | 168 | 126      | 4      | 117     | 16   | 19     |
| 40           | /Ocnyz  | 1439                | 168 | 40       | 1      | 36      | 11   | 11     |
| 41           | /Ocnzp  | 1439                | 168 | 49       | 3      | 45      | 24   | 9      |
| 42           | /Ocoxz  | 1439                | 168 | 112      | 5      | 14      | 3    | 2      |
| 43           | /Ocozp  | 1439                | 168 | 18       | 1      | 35      | 28   | 9      |
| 44           | /Oewez  | 720                 | 84  | 32       | 2      | 38      | 48   | 4      |
| 45           | /Oewnz  | 1439                | 168 | 52       | 3      | 20      | 15   | 1      |
| 46           | /Oewpp  | 1439                | 168 | 17       | 6      | 38      | 12   | 4      |
| 47           | /Oeyxp  | 720                 | 84  | 36       | 0      | 12      | 6    | 4      |
| 48           | 9DEeyz  | 180                 | 36  | 0        | 0      | 16      | 0    | 0      |
| 49           | 9DNeyz  | 30                  | 6   | 0        | 0      | 0       | 0    | 0      |
| 50           | 9DOeyz  | 360                 | 72  | 0        | 0      | 32      | 12   | 4      |
| 51           | 9DXeyz  | 720                 | 108 | 10       | 0      | 0       | 0    | 0      |
| 52           | 9DYeyz  | 720                 | 108 | 22       | 2      | 38      | 18   | 6      |
| 53           | 9DZlyz  | 1439                | 216 | 69       | 3      | 32      | 17   | 2      |
| 54           | 9DZepz  | 1439                | 216 | 71       | 7      | 32      | 16   | 7      |
| 55           | 9DZpez  | 1439                | 216 | 55       | 6      | 52      | 13   | 5      |
| 56           | 9DZppp  | 720                 | 108 | 54       | 12     | 72      | 18   | 20     |
| 57           | 9ODeyz  | 360                 | 36  | 4        | 0      | 4       | 0    | 0      |
| 58           | 9OFlyz  | 360                 | 36  | 76       | 0      | 72      | 8    | 4      |
| 59           | 9OFepz  | 720                 | 72  | 82       | 0      | 45      | 0    | 6      |
| 60           | 9OFppp  | 180                 | 18  | 208      | 0      | 134     | 8    | 0      |
| 61           | 9Oblyz  | 720                 | 72  | 54       | 4      | 41      | 12   | 2      |
| 62           | 9Obepz  | 1439                | 144 | 145      | 7      | 48      | 13   | 5      |
| 63           | 9Obmyz  | 1439                | 144 | 87       | 2      | 31      | 8    | 0      |
| 64           | 9Obnpz  | 720                 | 72  | 2        | 0      | 18      | 4    | 0      |
| 65           | 9Obofz  | 720                 | 72  | 68       | 4      | 28      | 2    | 0      |
| 66           | 9Obpez  | 720                 | 72  | 22       | 2      | 25      | 4    | 0      |
| 67           | 9Obpfp  | 720                 | 72  | 174      | 6      | 71      | 10   | 0      |
| 68           | 9Obppp  | 1439                | 144 | 30       | 3      | 26      | 5    | 4      |
| 69           | 9Odmppz | 720                 | 72  | 186      | 6      | 54      | 10   | 2      |
| 70           | 9Odmzp  | 360                 | 36  | 16       | 0      | 6       | 12   | 0      |
| 71           | 9Odoyz  | 360                 | 36  | 76       | 8      | 66      | 16   | 4      |
| 72           | 9Oewpp  | 360                 | 36  | 40       | 0      | 16      | 4    | 0      |
| 73           | DDDeyz  | 15                  | 1   | 0        | 0      | 0       | 0    | 0      |

| Isomorphisms |        | Number of Solutions |    |          |        |         |      |        |
|--------------|--------|---------------------|----|----------|--------|---------|------|--------|
| No.          | Name   | dicubes             |    | straight |        | L shape |      |        |
|              |        | 36                  | 30 | Edge     | Center | Corner  | Edge | Center |
| 74           | DDEeyz | 720                 | 36 | 2        | 2      | 2       | 2    | 0      |
| 75           | DDZQyz | 180                 | 7  | 0        | 0      | 0       | 0    | 0      |
| 76           | DDZlyz | 1439                | 56 | 1        | 3      | 5       | 0    | 1      |
| 77           | DDZofz | 360                 | 14 | 4        | 0      | 2       | 0    | 2      |
| 78           | DDZpez | 720                 | 28 | 0        | 4      | 4       | 0    | 0      |
| 79           | DDZppp | 360                 | 14 | 0        | 12     | 6       | 0    | 4      |
| 80           | DEDeyz | 360                 | 12 | 4        | 4      | 4       | 2    | 0      |
| 81           | DEFlyz | 1439                | 48 | 10       | 3      | 15      | 3    | 0      |
| 82           | DEFepz | 720                 | 24 | 18       | 6      | 16      | 0    | 0      |
| 83           | DEFppp | 360                 | 12 | 0        | 20     | 7       | 6    | 0      |
| 84           | DEOeyz | 720                 | 24 | 0        | 1      | 2       | 0    | 2      |
| 85           | DEPQyz | 720                 | 16 | 6        | 0      | 0       | 0    | 0      |
| 86           | DEPlyz | 1439                | 32 | 1        | 0      | 0       | 0    | 0      |
| 87           | DEPepz | 1439                | 32 | 8        | 3      | 3       | 1    | 2      |
| 88           | DEPmyz | 1439                | 32 | 11       | 2      | 1       | 0    | 1      |
| 89           | DEPnpz | 1439                | 32 | 23       | 5      | 3       | 0    | 1      |
| 90           | DEPofz | 1439                | 32 | 13       | 3      | 10      | 1    | 1      |
| 91           | DEPpez | 1439                | 32 | 17       | 2      | 17      | 1    | 3      |
| 92           | DEPpfp | 1439                | 32 | 2        | 1      | 0       | 0    | 0      |
| 93           | DEPppp | 1439                | 32 | 12       | 8      | 2       | 1    | 0      |
| 94           | DEZQyz | 1439                | 48 | 23       | 5      | 6       | 0    | 1      |
| 95           | DEZlyz | 360                 | 12 | 0        | 0      | 0       | 0    | 0      |
| 96           | DEZeyz | 1439                | 48 | 4        | 6      | 4       | 0    | 1      |
| 97           | DEZnpz | 720                 | 24 | 26       | 2      | 4       | 0    | 0      |
| 98           | DEZnyz | 720                 | 24 | 14       | 0      | 14      | 0    | 1      |
| 99           | DEZnzp | 720                 | 24 | 4        | 4      | 4       | 0    | 2      |
| 100          | DEZozp | 720                 | 24 | 10       | 14     | 6       | 0    | 3      |
| 101          | DEeOyz | 720                 | 16 | 6        | 2      | 0       | 0    | 0      |
| 102          | DEePpz | 1439                | 32 | 17       | 0      | 0       | 0    | 0      |
| 103          | DEeRez | 720                 | 16 | 44       | 0      | 0       | 0    | 0      |
| 104          | DEeRfp | 1439                | 32 | 16       | 0      | 0       | 0    | 0      |
| 105          | DEeYyz | 720                 | 16 | 0        | 0      | 0       | 0    | 0      |
| 106          | DEeZpz | 720                 | 16 | 22       | 0      | 2       | 0    | 0      |
| 107          | DEeZyz | 1439                | 32 | 8        | 3      | 6       | 0    | 0      |
| 108          | DEeZzp | 1439                | 32 | 4        | 0      | 0       | 0    | 0      |
| 109          | DEecpz | 1439                | 32 | 54       | 5      | 9       | 0    | 0      |
| 110          | DEecyz | 1439                | 32 | 39       | 3      | 48      | 3    | 0      |
| 111          | DEeczp | 1439                | 32 | 18       | 1      | 10      | 2    | 0      |

| Isomorphisms |         | Number of Solutions |    |          |        |         |      |        |
|--------------|---------|---------------------|----|----------|--------|---------|------|--------|
| No.          | Name    | dicubes             |    | straight |        | L shape |      |        |
|              |         | 36                  | 30 | Edge     | Center | Corner  | Edge | Center |
| 112          | DEedyz  | 1439                | 32 | 13       | 1      | 11      | 2    | 0      |
| 113          | DEeexz  | 1439                | 32 | 28       | 0      | 23      | 2    | 0      |
| 114          | DEeeyz  | 1439                | 32 | 15       | 1      | 2       | 0    | 0      |
| 115          | DEewez  | 1439                | 32 | 27       | 2      | 8       | 0    | 3      |
| 116          | DEewfp  | 1439                | 32 | 36       | 0      | 1       | 0    | 0      |
| 117          | DEewxp  | 1439                | 32 | 34       | 1      | 2       | 0    | 1      |
| 118          | DEexnz  | 1439                | 32 | 33       | 1      | 10      | 0    | 1      |
| 119          | DEexp   | 1439                | 32 | 31       | 1      | 47      | 1    | 1      |
| 120          | DEexyy  | 1439                | 32 | 47       | 0      | 3       | 1    | 1      |
| 121          | DZDQyz  | 90                  | 1  | 0        | 0      | 0       | 0    | 0      |
| 122          | DZDlyz  | 720                 | 8  | 2        | 2      | 6       | 0    | 0      |
| 123          | DZDofz  | 180                 | 2  | 8        | 0      | 0       | 0    | 0      |
| 124          | DZDpez  | 360                 | 4  | 0        | 0      | 12      | 1    | 0      |
| 125          | DZDppp  | 180                 | 2  | 0        | 8      | 6       | 0    | 2      |
| 126          | DZGOyz  | 1439                | 16 | 28       | 0      | 17      | 1    | 1      |
| 127          | DZDQfz  | 720                 | 8  | 4        | 0      | 2       | 0    | 0      |
| 128          | DZGRez  | 1439                | 16 | 20       | 0      | 11      | 0    | 0      |
| 129          | DZGRpp  | 720                 | 8  | 62       | 0      | 74      | 4    | 2      |
| 130          | DZGYyz  | 1439                | 16 | 15       | 3      | 9       | 2    | 0      |
| 131          | DZGZpz  | 1439                | 16 | 39       | 3      | 9       | 0    | 0      |
| 132          | DZGZzp  | 720                 | 8  | 4        | 2      | 6       | 0    | 0      |
| 133          | DZGlxz  | 720                 | 8  | 26       | 0      | 15      | 0    | 0      |
| 134          | DZGwez  | 1439                | 16 | 29       | 2      | 2       | 0    | 0      |
| 135          | DZGwnz  | 1439                | 16 | 36       | 0      | 8       | 0    | 0      |
| 136          | DZGwpp  | 720                 | 8  | 4        | 1      | 3       | 0    | 0      |
| 137          | DZGwxp  | 720                 | 8  | 45       | 0      | 15      | 0    | 0      |
| 138          | DZpO[z  | 180                 | 2  | 0        | 0      | 2       | 0    | 0      |
| 139          | DZpOez  | 360                 | 4  | 25       | 0      | 3       | 0    | 0      |
| 140          | DZpOnz  | 720                 | 8  | 200      | 6      | 22      | 0    | 1      |
| 141          | DZzOpp  | 1439                | 16 | 183      | 4      | 51      | 1    | 1      |
| 142          | DZpQnz  | 360                 | 4  | 8        | 0      | 0       | 0    | 0      |
| 143          | DZpQxp  | 720                 | 8  | 129      | 2      | 10      | 1    | 0      |
| 144          | DZpZpp  | 720                 | 8  | 172      | 6      | 61      | 2    | 0      |
| 145          | DZp\lp  | 360                 | 4  | 65       | 14     | 16      | 1    | 4      |
| 146          | DZp\lfp | 360                 | 4  | 24       | 6      | 12      | 4    | 1      |
| 147          | DZp\ly  | 360                 | 4  | 21       | 0      | 4       | 0    | 0      |
| 148          | Dzplpf  | 720                 | 8  | 137      | 0      | 69      | 1    | 1      |

### 1. Additional Comments

The Kolor Kraze puzzle shown in Reference 1 and the nine color puzzle shown in Reference 2 are isomorphic, thus leading me to believe that they originated from the same source.

It is not necessary that the tricube be straight, and, as mentioned in Section 2, an irregular tricube may have three locations. The number of solutions for these cases are included in Table 1.

There are two rather startling phenomena shown in Table 1. While some color combinations within the set of thirty dicubes have up to 720 isomorphisms, there are two color combinations (Numbers 73 and 121) which, no matter how the colors are swapped, map onto themselves. Verifying this would be an interesting exercise for the reader. Stranger yet, these two cases have no puzzle solutions. Is this a coincidence?

### 2. Permissible Locations of the Tricube as Determined by Parity Restrictions

|   |   |   |    |    |    |    |    |    |
|---|---|---|----|----|----|----|----|----|
| 1 | 2 | 3 | 10 | 11 | 12 | 19 | 20 | 21 |
| 4 | 5 | 6 | 13 | 14 | 15 | 22 | 23 | 24 |
| 7 | 8 | 9 | 16 | 17 | 18 | 25 | 26 | 27 |

Figure 2.

The three layers of the cube are checkered as shown in Figure 2, where odd numbers represent one state and even numbers the other. Each dicube when similarly checkered has one even and one odd number. Since the cube has fourteen odd numbers and thirteen even numbers, the tricube must fill two odd numbered spaces.

There are only two possibilities. The tricube location filling spaces 1, 10, and 19 is referred to as an “edge case” while the tricube location in spaces 5, 14, and 23 is referred to as a “center case.” Other possible locations are reflections or rotations of these and are not considered as separate puzzles solutions.

If an irregular tricube is used, there are three possible locations. The corner case fills spaces 1, 2, and 11; the edge case fills spaces 2, 5, and 11; the center case fills spaces 5, 11, and 14.

### 3. The “No Two in the Same Plane” Rule

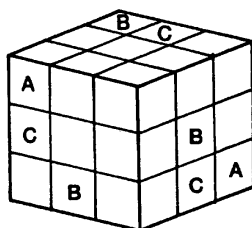


Figure 3.

One cube is centrally located and thus not visible. The only way the remaining two cubes of that color can be displayed on all six faces is for them to be located on opposite corners as shown in Figure 3 by the letter “A.”

The remaining six corners must have cubes of different colors. Since the corner colors are displayed on three faces, the remaining two cubes must be located on an edge, accounting for two faces, and on a centerface such as shown by the letter “B.” The remaining two colors must all be on edges, such as shown by the letter “C.”

In each case, cubes of the same color are never in the same plane.

### 4. Determination of the 133,105 Possible Dicube Color Combinations

The tricube colors are defined as colors 1, 2, and 3. The two dicubes with color 1 may use any two colors in the first row of the following table. If color 2 is selected from the first row, then a third dicube with color 2 and any color from the second row is selected. If color 2 is not selected from the first row then color 2 is combined with any two colors from the second row for the third and fourth dicubes.

If none of the previous selections include color 3, then color 3 is combined with two colors from the third row. If color 3 has been selected once, then one color is selected from the third row and if color 3 has been selected twice, then none are selected from the third row.

| First Color | Second Selection |
|-------------|------------------|
| 1           | 2 3 4 5 6 7 8 9  |
| 2           | 3 4 5 6 7 8 9    |
| 3           | 4 5 6 7 8 9      |
| 4           | 5 6 7 8 9        |
| 5           | 6 7 8 9          |
| 6           | 7 8 9            |
| 7           | 8 9              |
| 8           | 9                |

The process continues: color 4 is combined with colors in the fourth row such that there are three cubes with color 4, color 5 is combined with colors in the fifth row such that there are three cubes with color 5, etc.

Using this algorithm, a computer program established 133,105 possible color combinations.

### 5. Reduction of the Number of Isomorphisms

The table in Section 4 is simplified by restricting the selection in the first two rows as shown below.

| Color | Selection |
|-------|-----------|
| 1     | 2 3 4 5   |
| 2     | 3 4 5 6 7 |

This can be justified by defining the colors associated with color 1 that are not 2 or 3 as colors 4 and 5. Similarly if color 2 is not associated with colors 3, 4, or 5, the two new colors are identified as colors 6 and 7. This results in requiring only thirty cubes and, using the same algorithm as before, only 10,691 color combinations.

### 6. Identification and Separation into Disjoint Sets

The color combinations in Figure 1 use dicubes with color (1, 2), (1, 4), (2, 3), (3, 5), (4, 6), (4, 7), (5, 6), (5, 8), (6, 9), (7, 8), (7, 9) and (8, 9).

Identification of the second color of these ordered pairs is sufficient to define all the colors. For example, given the second numbers 2, 4, 3, 5, 6, 7, 6, 8, 9, 8, 9, and 9 one can logically deduce the first colors by the

three cubes of each color requirement and the restriction that the second number be greater than the first.

Identification was further simplified by placing these numbers in pairs (see Note below), i.e., (24), (35), (67), (68), (98), and (99), and then assigning a printable character to each, by adding twenty-three to each and using the ASCII character set codes (ASCII stands for American Standard Code for Information Interchange). Thus this color combination is identified as `/:Z[yz`.

The use of this six-character identification greatly simplified the process of sorting the 10,691 cases into 148 disjoint sets of isomorphisms.

Decoding a name is quite simple. For example, the first entry in Table 1 is `/:Z[yz`. The ASCII codes for these characters are: 46, 58, 90, 91, 121, and 122. Subtracting twenty-three results in: 23, 35, 67, 68, 98, and 99. It is then a simple process to recognize that this represents the dicubes colored (1, 2), (1, 3), (2, 3), (4, 5), (4, 6), (4, 7), (5, 6), (5, 8), (6, 9), (7, 8), (7, 9), and (8, 9).

Any permissible interchange of colors is an isomorphism. Color 2, which is in the center of the tricube, cannot be interchanged with other colors. Colors 1 and 3, on the ends of the tricube, can be swapped with each other, but not with any of the other colors. The remaining six colors may be interchanged in numerous ways: two at a time, three at a time, four at a time, including pairs of two at a time, five at a time including three at a time with two at a time, and six at a time including a triplet of two at a time, pairs of three at a time and four at a time with two at a time. Not all of these mappings produce a new isomorphism.

**Note:** The smallest possible number is 23 and the largest is 99. The ASCII code for 23 is not a printable character. An inspection of the ASCII table will explain why it was decided to add 23.

The 10,691 color combinations obtained by the algorithm described in Section 5 were listed in order according to their ASCII characters. The first entry, `/:Z[yz`, has sixty isomorphisms. These were removed from the list. The head of the list then became `/:Zppp`; its isomorphisms are determined and removed from the list. The process was then continued until the list was exhausted. This process then identified the 148 disjoint sets of isomorphisms.

Shown in Figure 4 are an edge solution and a center solution for the color combination of Figure 1. The solutions are identified by a three-digit number indicating the color of the buried cube and the two "edge" colors. Different solutions often have the same three-digit identification.

|   |   |   |            |   |   |   |   |   |   |   |   |            |   |   |   |   |   |
|---|---|---|------------|---|---|---|---|---|---|---|---|------------|---|---|---|---|---|
| 1 | 8 | 5 | 2          | 7 | 9 | 3 | 4 | 6 | 3 | 4 | 6 | 1          | 7 | 9 | 2 | 8 | 5 |
| 4 | 6 | 3 | 5          | 1 | 8 | 9 | 2 | 7 | 5 | 1 | 7 | 6          | 2 | 8 | 9 | 3 | 4 |
| 7 | 9 | 2 | 6          | 3 | 4 | 8 | 5 | 1 | 8 | 9 | 2 | 4          | 5 | 3 | 7 | 6 | 1 |
|   |   |   | <b>149</b> |   |   |   |   |   |   |   |   | <b>249</b> |   |   |   |   |   |

**Figure 4.** Typical solutions.

If the solutions are to be catalogued, then additional criteria for recording solutions are recommended.

The edge solution is interesting in that new solutions are often obtainable by using two transformations. Whenever the tricube is on an edge and shares a plane with only three dicubes, the plane can be translated to the other side resulting in a new solution. Often two dicubes may be swapped with two others having the same colors. For example, dicubes (4, 7) and (6, 9) in the bottom layer may be swapped with dicubes (4, 6) and (7, 9) on the upper right. Using these transformations, the reader should now be able to determine five more edge solutions.

Stan Isaacs has suggested that graph analysis would be useful in determining the number of solutions. A preliminary investigation shows some merit in utilizing graph analysis to illustrate why some color combinations have no solutions while other color combinations have numerous solutions. Unfortunately, all 148 graphs have not been compared.

Puzzle sets or puzzle solutions may be obtained by contacting the author by E-mail at [sivy@ieee.org](mailto:sivy@ieee.org).

## References

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